# Principles of Communications

## EES 351


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## 4.2 Energy and Power

**Office Hours:**

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# Review: Energy and Power

- Consider a signal $g(t)$.
- Total (normalized) **energy**:

Parseval's Theorem [2.43]

[Defn. 4.13] $$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \lim_{T\to\infty}\int_{-T}^{T} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df.$$

- Average (normalized) **power**:

[Defn. 4.15] $$P_g = \left\langle |g(t)|^2 \right\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2 dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} |g(t)|^2 dt.$$

time-average operator
[Defn. 4.16a]

# Review: Time average vs. Inner Product

Inner Product:

$$\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt$$

two arguments

Time Average:

$$\langle g(t) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)\, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} g(t)\, dt.$$

one argument

# Power Calculation

| | $g(t)$ | $P_g = \langle \lvert g(t) \rvert^2 \rangle$ |
|---|---|---|
| [4.18] | Periodic with period $T_0$ | $\frac{1}{T_0} \int_{T_0} \lvert g(t) \rvert^2 dt$ |
| Linear combination of complex exponential functions [4.23] | $\sum_k c_k e^{j2\pi f_k t}$ where the $f_k$ are distinct | $\sum_k \lvert c_k \rvert^2$ |

# Power Calculation: Special Cases

| | $g(t)$ | $P_g = \langle \lvert g(t) \rvert^2 \rangle$ |
|---|---|---|
| Linear combination of complex exponential functions [4.23] | $\sum_k c_k e^{j2\pi f_k t}$ where the $f_k$ are distinct | $\sum_k \lvert c_k \rvert^2$ |
| Linear combination of sinusoids [4.28] | $\sum_k A_k \cos(2\pi f_k t + \phi_k)$ where the $f_k$ are positive and distinct | $\frac{1}{2} \sum_k \lvert A_k \rvert^2$ |

# Summary (1)

- **(Total) Energy:** $E_g = \int_{-\infty}^{\infty} |g(t)|^2\, dt = \int_{-\infty}^{\infty} |G(f)|^2\, df$
- **Average Power:** $P_g = \langle |g(t)|^2 \rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |g(t)|^2\, dt$ "energy per unit time"
  - For periodic signal: $P_g = \frac{1}{T_0}\int_{T_0} |g(t)|^2\, dt = \frac{\text{energy in one period}}{\text{period}}$
  - Other special cases:

| | $g(t)$ | $P_g = \langle \lvert g(t)\rvert^2 \rangle$ |
|---|---|---|
| Linear combination of complex exponential functions [4.23] | $\sum_k c_k e^{j2\pi f_k t}$ where the $f_k$ are distinct | $\sum_k \lvert c_k\rvert^2$ |
| Linear combination of sinusoids [4.28] | $\sum_k A_k \cos(2\pi f_k t + \phi_k)$ where the $f_k$ are positive and distinct | $\frac{1}{2}\sum_k \lvert A_k\rvert^2$ |

- **Time Average:** $\langle g(t) \rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} g(t)\, dt$
  - For periodic signal: $\langle g(t) \rangle = \frac{1}{T_0}\int_{T_0} g(t)\, dt$

# Summary (2)

- **(Total) Energy:** $E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt = \int_{-\infty}^{\infty} |G(f)|^2 \, df$
  - A signal $g(t)$ is an **energy signal** if $0 < E_g < \infty$.
    - Any energy signal $g(t)$ has $P_g = 0$.
- **Average Power:** $P_g = \langle |g(t)|^2 \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 \, dt$ ("energy per unit time")
  - A signal $g(t)$ is a **power signal** if $0 < P_g < \infty$.
    - Any power signal $g(t)$ has $E_g = \infty$.
  - For periodic signal:

    $$P_g = \frac{1}{T_0} \int_{T_0} |g(t)|^2 \, dt = \frac{\text{energy in one period}}{\text{period}}$$
  - Other special cases:

| | $g(t)$ | $P_g = \langle \lvert g(t)\rvert^2 \rangle$ |
|---|---|---|
| Linear combination of complex exponential functions [4.23] | $\sum_k c_k e^{j2\pi f_k t}$ where the $f_k$ are distinct | $\sum_k \lvert c_k\rvert^2$ |
| Linear combination of sinusoids [4.28] | $\sum_k A_k \cos(2\pi f_k t + \phi_k)$ where the $f_k$ are positive and distinct | $\frac{1}{2}\sum_k \lvert A_k\rvert^2$ |

# Summary (3)

- **(Total) Energy:** $E_g = \int_{-\infty}^{\infty} |g(t)|^2\, dt = \int_{-\infty}^{\infty} |G(f)|^2\, df$
  - A signal $g(t)$ is an **energy signal** if $0 < E_g < \infty$.
    - Any energy signal $g(t)$ has $P_g = 0$.
- **Average Power:** $P_g = \langle |g(t)|^2 \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2\, dt$

"energy per unit time"

**Time Average:**
$$\langle g(t) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} g(t)\, dt$$

For periodic signal:
$$\langle g(t) \rangle = \frac{1}{T_0} \int_{T_0} g(t)\, dt$$

  - A signal $g(t)$ is a **power signal** if $0 < P_g < \infty$.
    - Any power signal $g(t)$ has $E_g = \infty$.
  - For periodic signal:
    $$P_g = \frac{1}{T_0} \int_{T_0} |g(t)|^2\, dt = \frac{\text{energy in one period}}{\text{period}}$$
  - Other special cases:

| | $g(t)$ | $P_g = \langle \lvert g(t) \rvert^2 \rangle$ |
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| Linear combination of complex exponential functions [4.23] | $\sum_k c_k e^{j2\pi f_k t}$ where the $f_k$ are distinct | $\sum_k \lvert c_k \rvert^2$ |
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